Nonlinear modulation of Rabi oscillations in a one-dimensional nonlinear periodic photonic structure

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We study nonlinear dynamics of classical electromagnetic wave propagation in a one-dimensional nonlinear periodic photonic structure. It is found that the period of Rabi oscillations can be modulated by the relatively weak nonlinearity ($2V_0/\gamma>1$). When nonlinearity is relatively strong compared to the strength of resonant coupling ($2V_0/\gamma<1$), Rabi oscillations is suppressed and the system shows a dynamical behavior, i.e., energy localizes in one mode rather than full oscillation between two degenerated modes. Phase plane analysis is applied to explain these dynamical phenomena.

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Landau-Zener tunneling [1] and Bloch oscillations [2] are two interesting macroscopic quantum phenomena in quantum physics, performed by classical electromagnetic wave propagation in one-dimensional periodic structures based on waveguide arrays and superlattices [3–7] or in the two-dimensional case [8], and described by the Landau-Zener-Majorana model [9–14]. Rabi oscillation, another quantum phenomenon, was first reported by Shchesnovich and Chavez-Cerda [15] with classical wave propagation in a periodic photonic lattice, where they showed that optical beam propagation induced Rabi oscillations between Fourier spectral peaks, i.e., the beam position oscillations in the spatial coordinates, satisfying Bragg resonance conditions.

However, nonlinear effects of this system have not been taken into consideration so far. Therefore, it is of great value to investigate influences of nonlinear effects on the dynamic behaviors of classical wave propagation. In this paper, the propagation of classical wave in a one-dimensional nonlinear periodic photonic structure is investigated and result in Rabi oscillations between two degenerated modes and most of energy localizes in one mode, which is similar to self-trapping of Bose-Einstein condensation in one state [16–18]. We apply phase plane analysis theory to explain these phenomena in such a classical system.

The propagation of classical electromagnetic wave in a one-dimensional nonlinear periodic photonic structure, studied in this paper, may be created by optically induced or periodic waveguide arrays [19–21]. It is described by the nonlinear Schrödinger equation:

$$i\frac{\partial E}{\partial z} = -\frac{1}{2}\frac{\partial^2}{\partial z}E + V(x)E - \gamma|E|^2E, \tag{1}$$

where E is the dimensionless electric field amplitude, z,x are the propagation and transverse coordinate, respectively, and γ is the nonlinear coefficient. The potential $V(x)$ selected here is in the form of a one-dimensional periodic photonic lattice, i.e.,

$$V(x) = V_0 \cos x. \tag{2}$$

Furthermore, the Bragg resonance considered here is at the edge of the first Brillouin zone. The resonant part of field E can be described as

$$E = C_1(z)e^{ik_Bx} + C_2(z)e^{-ik_Bx}, \tag{3}$$

with $k_B=1/2$.

Substituting Eqs. (2) and (3) into Eq. (1), the incident amplitude C_1 and the Bragg reflected amplitude C_2 can be estimated by the following resonant coupling equations:

$$i\frac{\partial C_1}{\partial z} = [\beta - \gamma(|C_1|^2 + 2|C_2|^2)]C_1 + \frac{V_0}{2}C_2,$$

$$i\frac{\partial C_2}{\partial z} = [\beta - \gamma(2|C_1|^2 + |C_2|^2)]C_2 + \frac{V_0}{2}C_1, \tag{4}$$

with $\beta=k_B^2/2$.

Some dynamical variables (R and F) and the difference in distribution of classical wave (M) between two resonant states are defined as follows:

$$M = |C_1|^2 - |C_2|^2,$$

$$R = \frac{1}{2}(C_1^*C_2 + C_2^*C_1),$$

$$F = \frac{1}{2}(C_1^*C_2 - C_2^*C_1) \tag{5}$$

Substituting Eq. (4) into Eq. (5), it is found that the system mentioned here is analogous to that of classical wave propagation in the nonlinear photonic structure:

$$\dot{M} = 2V_0F,$$

$$\dot{R} = -\gamma MF,$$

$$\dot{F} = \gamma MR - \frac{V_0}{2}M. \tag{6}$$

According to the above equations, the dynamics of this system are determined by the competition between the nonlinearity and the resonant coupling interactions, i.e., different strength of competition may lead to different dynamical behaviors of the system.

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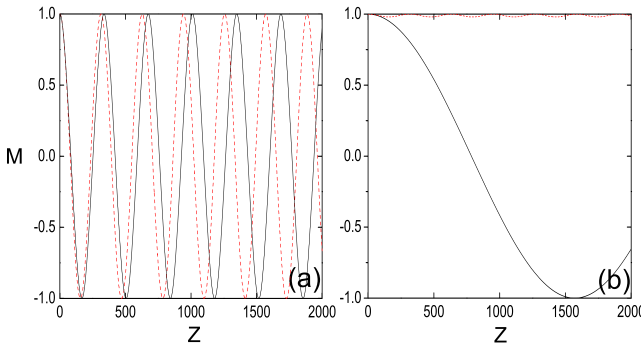

FIG. 1. (Color online) Initial condition is $M(0)=1$ and $R(0)=F(0)=0$. (a) Black solid line with $V_0=0.02$, $\gamma=0$ and red dash line with $V_0=\gamma=0.02$. (b) Black solid line with $V_0=0.02$, $\gamma=0$ and red dash line with $V_0=0.002$, $\gamma=0.02$.

In the spatial coordinates, the classical wave position is in the form of $\langle x\rangle=\int dxx|E(x,z)|^2/\int dx|E(x,z)|^2$. Furthermore, it is assumed that the initial condition considered in this paper is $E(x,0)=\exp(ik_Bx)$;

$$\frac{d\langle x\rangle}{dz}=V_0(|C_1|^2-|C_2|^2)=V_0M. \qquad (7)$$

Equation (6) can also be analytical in terms of the Jacobi elliptic function with the oscillation period given by

$$T=2K\left(\frac{2V_0}{\gamma}\right)/\gamma, \quad 2V_0<\gamma,$$

$$T=4K\left(\frac{\gamma}{2V_0}\right)/V_0, \quad 2V_0>\gamma, \qquad (8)$$

where $K(.)$ is the complete elliptic integral of the first kind.

According to the definition of R and F, it is easy to understand that $R^2+F^2=(1-M^2)/4$. With $\tan\phi=F/R$, one may find

$$\dot{M}=V_0\sqrt{1-M^2}\sin\phi,$$

$$\dot{\phi}=\gamma M-\frac{V_0M}{\sqrt{1-M^2}}\cos\phi. \qquad (9)$$

It is interesting to note that the above equations can be mapped into a classical Hamiltonian by treating M and ϕ as canonically conjugate variables, i.e., $\partial\phi/\partial z=\partial H/\partial M$, $\partial M/\partial z=-\partial H/\partial\phi$,

$$H=\frac{\gamma}{2}M^2+V_0\sqrt{1-M^2}\cos\phi. \qquad (10)$$

It describes a nonrigid or momentum-shorted pendulum which length is proportional to $\sqrt{1-M^2}$.

Dynamical behaviors of classical wave propagating in nonlinear photonic structure are shown in Fig. 1. Without nonlinearity, energy oscillates between two degenerated modes (Rabi oscillations), the period of which becomes longer with the decrease in the strength of resonant coupling (V_0), just as the black solid lines of Fig. 1. Taking nonlinearity into account, the system may exhibit different dynamical behaviors. When the nonlinearity compared to the resonant coupling strength is relatively weak, i.e., $2V_0/\gamma>1$, the system also only exhibits Rabi oscillations, but the period of which becomes shorter compared to that without nonlinearity [the red dash line of Fig. 1(a)]. Therefore, the period of the Rabi oscillations can be modulated by the relatively weak nonlinear effects. When nonlinearity compared to the strength of resonant coupling is relatively strong, i.e., $2V_0/\gamma<1$, energy localizes mostly in one mode rather than full oscillates between two degenerated modes [the red dash line of Fig. 1(b)]. It means that the average difference in distribution of classical waves between these two resonant states is nonzero, i.e., $\langle M\rangle\neq 0$.

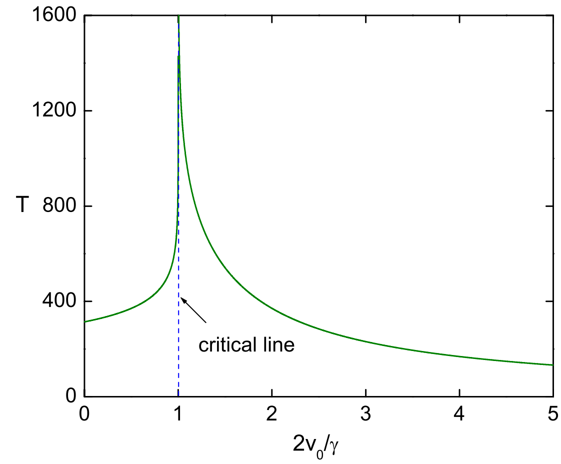

FIG. 2. (Color online) Dependence of the period to the $2V_0/\gamma$.

Figure 2 demonstrates the oscillation period versus the $2V_0/\gamma$. When nonlinearity is relatively weak, i.e., $2V_0/\gamma>1$, the equivalent pendulum assumes a librator rotation—a full oscillation of M (Rabi oscillations). But if the nonlinearity compared to the resonant coupling strength is relatively strong, i.e., $2V_0/\gamma<1$, the gradually increasing period resembles that of nonlinear single pendulum oscillation, where energy localizes mostly in one mode. In a word, the competition between nonlinearity and resonant coupling strength can modulate the dynamical behaviors of the system, i.e., we can well control propagating properties of classical wave in this system (energy oscillation between two degenerated modes or localization most in one mode) by selecting appropriate nonlinear and resonant coupling parameters.

Phase plane analysis of the system with classical Hamiltonian [Eq. (10)] is applied to describe the above dynamical properties of classical wave propagation in such a nonlinear periodic photonic structure.

For $2V_0/\gamma>1$, as shown in Fig. 3(a), there are two fixed points in the phase plane, marked P1 with $M=0,\phi=\pm\pi$ and P2 with $M=0,\phi=0$, respectively. In this case, the classical wave distribution in both resonant states is in equilibrium. Trajectories around the fixed points, shown in Fig. 3(a), represent full oscillations of classical wave between two resonant states corresponding to different initial conditions. The red curve of Fig. 3(a) is the evolution of our classical wave system, i.e., the system with initial conditions $M(0)=1$, $R(0)=F(0)=0$, and $\phi=0$. Therefore, the average difference

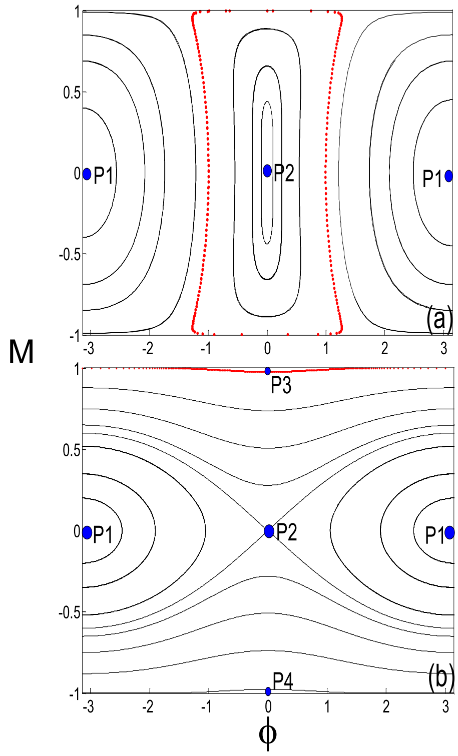

FIG. 3. (Color online) Trajectories on the phase plane of Hamiltonian (10). (a) $V_0=\gamma=0.02$ and $2V_0/\gamma=2$. (b) $V_0=0.002$, $\gamma=0.02$, and $2V_0/\gamma=0.2$. Red curves correspond to the evolution our classical wave system [initial condition is $(M(0)=1$, $R(0)=F(0)=0$, and $\phi=0]$.

in distribution of classical wave between two resonant states is zero, i.e., $\langle M\rangle=0$, which indicates Rabi oscillations of the system and is consistent with the result of our numerical simulation—the green line of Fig. 4.

For $2V_0/\gamma<1$, there are two more fixed points on the line $\phi=0$ marked P3 and P4 [Fig. 3(b)]. They located at $M=-m,m$, respectively, with which $m=\sqrt{1-(V_0/\gamma)^2}$. In this case, it is found that the distribution of classical wave corresponding to each resonant state is not in equilibrium at the

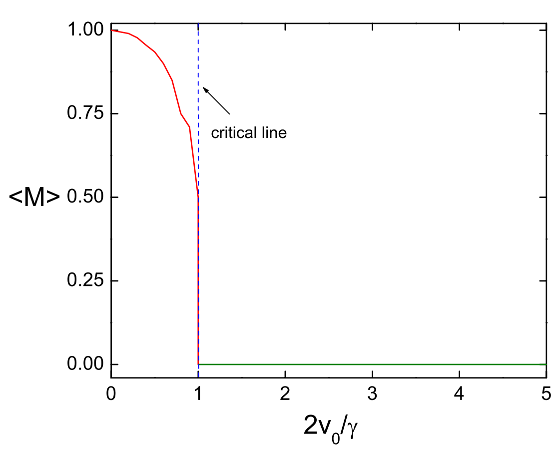

FIG. 4. (Color online) The average of classical wave distribution difference between two Bragg resonant states when the trajectory $M(0)=1$ and $\phi=0$.

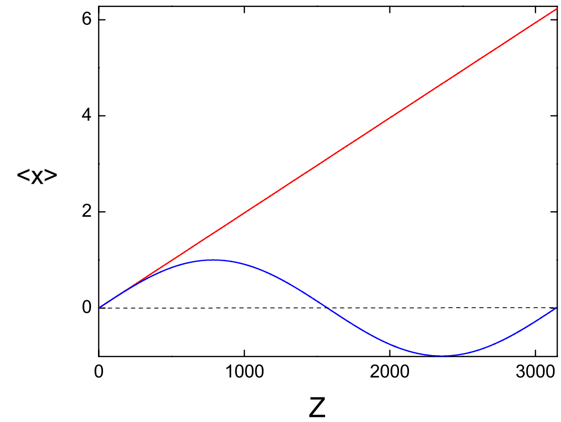

FIG. 5. (Color online) Average position of classical wave correspond to Fig. 1. Blue line with $V_0=0.02$, $\gamma=0$ and red line with $V_0=0.002$, $\gamma=0.02$.

point P3 or P4 (P3 and P4 can be considered as the center of two discrete breathers.). Just as the red curve of Fig. 3(b) shown, the evolution of M in our classical wave system is around P3. Therefore, the average difference in distribution of classical wave in both resonant states is nonzero, i.e., $\langle M\rangle\neq 0$, which indicates that classical wave is localized in one resonant state and agrees with the result of our numerical simulation shown in Fig. 4 (red line).

It should be noted that energy of classical wave localizes mostly in one resonant state for $2V_0/\gamma<1$, as shown in the red dash line of Fig. 1(b) and the red curve of Fig. 3(b). In this case, it corresponds to the formation of the soliton and can also be considered as the formation of a state with asymmetrical energy distribution, where nonlinearity plays a leading role.

Figure 5 demonstrates classical wave in the spatial coordinate classical wave oscillates around $\langle x\rangle=0$ without nonlinearity (the blue line of Fig. 5). When nonlinearity is relatively strong compared to the strength of resonant coupling, such as $2V_0/\gamma=0.2$, the classical wave propagates only in one direction without any oscillations (the red line of Fig. 5). Based on the above analysis, we conclude that nonlinearity can suppress position oscillations of classical wave in the spatial coordinates.

In conclusion, we have studied propagating properties of classical wave in a one-dimensional nonlinear periodic photonic structure by taking the nonlinearity into account. Nonlinear modulation on the period of Rabi oscillations has been found for relatively weak nonlinearity $(2V_0/\gamma>1)$. When nonlinearity compared to the strength of resonant coupling is relatively strong, i.e., $2V_0/\gamma<1$, the Rabi oscillation is suppressed and the system shows a dynamical behavior, i.e., energy localization most in one mode rather than full oscillation between two generated modes. We employ phase plane analysis to well describe these dynamical behaviors. It is of great value to conclude that we can control the dynamics of the system by selecting appropriate nonlinear and resonant coupling parameters.

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